Landau diamagnetism

We consider a gas of electrons in a magnetic field. The dispersion is given by

$$\varepsilon(k_z) = \frac{k_z^2}{2m} + \left(n+\frac{1}{2}\right)\omega_c \quad , \quad n = 0, 1, 2, \ldots$$

$$\omega_c = \frac{|e|H}{mc}$$

The number of states in an interval dk_z of momenta is

$$2\,\frac{V|e|H}{(2\pi)^2 c}\,dk_z = \frac{V|e|H}{2\pi^2 c}\,dk_z$$

The grand potential

$$\Omega = -T\sum_{k} \ln\left(1+e^{\frac{\mu-\varepsilon_k}{T}}\right)$$

$$\Omega = -T\,\frac{V|e|H}{2\pi^2 c}\int_{-\infty}^{+\infty} \ln\left[1+e^{\frac{\mu}{T}-\frac{k_z^2}{2mT}-\frac{\omega_c(n+\frac{1}{2})}{T}}\right] dk_z$$

may be rewritten as

$$\Omega = \omega_c \sum_{n=0}^{\infty} f\left[\mu-\left(n+\tfrac{1}{2}\right)\omega_c\right]$$

where $f(\mu) = -\frac{TmV}{2\pi^2}\int_{-\infty}^{+\infty} \ln\left[1+e^{\frac{\mu}{T}-\frac{k_z^2}{2mT}}\right] dk_z$

Use a variation of Euler-Maclaurin formula

$$\sum_{n=0}^{\infty} F\left(n+\tfrac{1}{2}\right) \approx \int_0^{\infty} F(x)\,dx + \frac{1}{24}F'(0)$$

A more conventional form of the Euler-Maclaurin formula is

$$\sum^{\infty} F(n+a) \approx \int^{\infty} F(x)\,dx - \frac{1}{12}F'(a)$$

Maclaurin formula is

$$\frac{1}{2}F(a) + \sum_{n=1}^{\infty} F(n+a) \approx \int_0^{\infty} F(x)\,dx - \frac{1}{12}F'(a)$$

To arrive at the above formula, set $a = \frac{1}{2}$ and expand $F(x) \approx F(0) + x F'(0)$

Then $\Omega = \omega_c H \int_0^{\infty} f(\mu - \omega_c x)\,dx + \frac{\omega_c}{24} \left.\frac{\partial f(\mu - n\omega_c)}{\partial n}\right|_{n=0} =$

$$= \underbrace{\int_{-\infty}^{\mu} f(x)\,dx}_{\text{This is } \Omega \text{ at } H=0} - \frac{\omega_c^2}{24}\frac{\partial f(\mu)}{\partial \mu}$$

Thus, $\boxed{\Omega = \Omega_0(\mu) - \frac{1}{6}\tilde{\mu}_B^2 H^2 \frac{\partial^2 \Omega_0(\mu)}{\partial \mu^2}}$

where $\tilde{\mu}_B = \frac{|e|}{2mc}$ „Bohr" magneton with the electron mass replaced by the effective mass

Reminder: $\Omega = -\frac{(3\pi^2)^{\frac{2}{3}}}{5}\frac{1}{m}\left(\frac{N}{V}\right)^{\frac{2}{3}} N$

The magnetisation

$$\vec{M} = -\left(\frac{\partial \Omega}{\partial \vec{H}}\right)_{T,V,\mu}, \quad \chi = \frac{\partial M}{\partial H}$$

Diamagnetism $\longleftrightarrow$ $\vec{M}$ antiparallel to $\vec{H}$

$$\boxed{\chi_{dia} = \frac{\tilde{\mu}_B^2}{V}\frac{\partial^2 \Omega_0}{\partial \mu^2}}$$